1. Let $A = \{1, 2, 3\}$ and $B = \{x, y\}$.

i) List all subsets of $B$: $\mathcal{P}(B) =$

ii) Write two elements of the Cartesian product $A \times \mathcal{P}(B)$.

iii) How many elements are in $A \times \mathcal{P}(B)$?

iv) Define a mapping from $A$ into $A$ which is not 1-1.

v) How many mappings are there from $B$ to $A$?

vi) How many 1-1 mappings are there from $B$ to $A$?

vii) Does $P = \{\{1, 2\}, \{2, 3\}, \{3\}\}$ form a partition for $A$?

viii) How many relations can be defined on $B$?

ix) Define a relation on $B$ that has the symmetric property but does not have the reflexive property or the transitive property.

x) Define a relation on $B$ that has the reflexive and transitive properties but does not have the symmetric property.

xi) Define an equivalence relation on $A$ and exhibit all the different equivalence classes.

Let $A = \{1, 3, 11\}$ and let $R = \{(1, 1), (3, 11), (11, 3)\}$. Which of the following is not a correct statement?

i) $R$ is a subset of $A \times A$.  
\[
\text{ii) } R \text{ defines a relation on } A. 
\]

\[
\text{ii) } R \text{ defines a mapping from } A \text{ into } A. 
\]

\[
\text{iv) } R \text{ defines a 1-1 mapping (function) from } A \text{ into } A. 
\]

\[
\text{v) } R \text{ defines an onto mapping from } A \text{ into } A. 
\]

\[
\text{vi) } R \text{ has the reflexive property.} 
\]

\[
\text{vii) } R \text{ has the symmetric property.} 
\]

\[
\text{viii) } R \text{ has the transitive property.} 
\]

\[
\text{ix) } A \text{ is a proper subset of } R. 
\]
2. Define a binary operation "⊙" on \( \mathbb{Z} \), the set of integers, by \( a \odot b = a + b - ab, \ a, b \in \mathbb{Z} \). 

Is \( \mathbb{Z} \) closed under \( \odot \)? ii) Is \( \odot \) commutative? iii) Is \( \odot \) associative? iv) What is \( 5 \odot (-6) \)?

3. Let \( A = \{1, 2, 3\} \). Consider the following subsets of \( A \times A \):

I) \( \{(1,1),(2,2),(3,3)\} \)  
II) \( \{(1,1),(2,3),(3,2),(3,3),(2,2)\} \)  
III) \( \{(1,1),(2,3),(3,2)\} \)  
IV) \( \{(1,1),(1,3),(3,1),(3,3),(2,2)\} \)  
V) \( \{(3,3)\} \)

Which of the above defines:

i) a relation on \( A \). 
ii) a mapping (function) from \( A \) into \( A \). 
iii) a 1-1 mapping (function) from \( A \) into \( A \). 
iv) an onto mapping (function) from \( A \) into \( A \). 
v) a relation on \( A \). vi) a reflexive relation on \( A \).  
vii) a symmetric relation on \( A \). viii) a transitive relation on \( A \). ix) an equivalence relation on \( A \). x) a single element(singleton) .

5. Determine whether the following mappings are 1-1 and/or onto.

i) \( \alpha: \mathbb{Z} \to \mathbb{Z} \); \( \alpha(n) = 4n, n \in \mathbb{Z} \).

Find the image of \( \mathbb{Z} \) under \( \alpha \), \( \alpha(\mathbb{Z}) = \). 

Is \( \alpha \) 1-1? 

ii) \( \gamma: \mathbb{Z} \to \mathbb{Z} \); \( \gamma(n) = n^2, n \in \mathbb{Z} \), find \( \gamma(\mathbb{Z}) = \). 

Is \( \gamma \) 1-1?  
iii) \( \beta: \mathbb{Z} \to \mathbb{Z}; \ \beta(n) = \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ \frac{n-1}{2} & \text{if } n \text{ is odd} \end{cases} \), find \( \beta((\mathbb{Z}) = \) ________________________.

Is \( \beta \) 1-1 and/or onto? ____________________________.

iv) \( \gamma: \mathbb{Z} \times \mathbb{Z} \to \mathbb{R}; \ \gamma(n,m) = 2^n3^m, (n,m) \in \mathbb{Z} \times \mathbb{Z}. \)

Is \( \gamma \) 1-1 and/or onto? ____________________________.

Find \( \gamma(4,0) = \) ____________________________.

6. Let \( \mathbb{Z} \) be the set of all integers. For \( a, b \in \mathbb{Z} \), let us define \( a \sim b \) to mean that \( a - b \) is a multiple of 6.

i) Verify that "\( \sim \)" is an equivalence relation on \( \mathbb{Z} \).

v) Determine the equivalence classes \([5],[0],[−4]\).

How many different equivalent classes do you get?

7. Prove or disprove that \( 3|a(2a^2 + 7) \) for every integer \( a \).

8. Prove that if \( gcd(a, b) = 1 \), then \( gcd(a^2, b) = 1 \).

9. Find \( d \), the \( gcd(a, b) \) where \( a = 420 \) and \( b = 240 \).

Now find integers \( r \) and \( s \) such that \( ar + bs = d \)

10. Let \( \mathbb{R} \) be the set of all real numbers. For \( a, b \in \mathbb{R} \), let us define \( a \sim b \) to mean that \( a - b \) is an integer.

i) Verify that "\( \sim \)" is an equivalence relation on \( \mathbb{R} \).

ii) Determine the equivalence classes \([-10], [-\sqrt{2}], [\sqrt{2}], \) and \( \left[ \frac{3}{2} \right] \).