Section 2.4

Applications
Learning Objectives

1. Correctly interpret a bearing.

2. Solve a real-life problem involving an angle of elevation or depression.

3. Solve a real-life problem involving bearing.

4. Solve an applied problem using right triangle trigonometry.
Example 1

The two equal sides of an isosceles triangle are each 24 centimeters. If each of the two equal angles measures $52^\circ$, find the length of the base and the altitude.

Solution:

An isosceles triangle is any triangle with two equal sides. The angles opposite the two equal sides are called the base angles, and they are always equal.

Figure 1 shows a picture of our isosceles triangle.
We have labeled the altitude $x$. We can solve for $x$ using a sine ratio.

\[
\text{If } \quad \sin 52^{\circ} = \frac{x}{24}
\]

\[
\text{then } \quad x = 24 \sin 52^{\circ} = 24(0.7880) = 19 \text{ cm}
\]

Rounded to two significant digits

We have labeled half the base with $y$. To solve for $y$, we can use a cosine ratio.

\[
\text{If } \quad \cos 52^{\circ} = \frac{y}{24}
\]

\[
\text{then } \quad y = 24 \cos 52^{\circ} = 24(0.6157) = 15 \text{ cm}
\]

To two significant digits

The base is $2y = 2(15) = 30 \text{ cm}$. 
For some applications, we need the following definition.

**Definition**

An angle measured from the horizontal up is called an *angle of elevation*. An angle measured from the horizontal down is called an *angle of depression* (Figure 2).

These angles of elevation and depression are always considered positive angles.
Example 2
If a 75.0-foot flagpole casts a shadow 43.0 feet long, to the nearest 10 minutes what is the angle of elevation of the sun from the tip of the shadow?

Solution:
We begin by making a diagram of the situation. If we let $\theta$ = the angle of elevation of the sun, then

$$\tan \theta = \frac{75.0}{43.0}$$

$\tan \theta = 1.7442$

$$\theta = \tan^{-1} (1.7442) = 60^\circ \: 10'$$

Also see example 3 on p 86 of the e-book.
Our next applications are concerned with what is called the *bearing of a line*. It is used in navigation and surveying.

**DEFINITION**

The *bearing of a line* $l$ is the acute angle formed by the north–south line and the line $l$. The notation used to designate the bearing of a line begins with N or S (for north or south), followed by the number of degrees in the angle, and ends with E or W (for east or west).
Example 5

San Luis Obispo, California, is 12 miles due north of Grover Beach. If Arroyo Grande is 4.6 miles due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?

Solution:
We are looking for the bearing of San Luis Obispo from Arroyo Grande, so we will put our N-S-E-W system on Arroyo Grande (Figure 8).
We have known that when two parallel lines are crossed by a transversal, alternate interior angles are equal.

Now we can solve for $\theta$ using the tangent ratio.

\[
\tan \theta = \frac{4.6}{12} \quad \Rightarrow \quad \tan \theta = 0.3833
\]

\[
\theta = \tan^{-1} (0.3833) = 21^\circ \quad \text{To the nearest degree}
\]

The bearing of San Luis Obispo from Arroyo Grande is N 21° W.

See examples 6 and 7